B. Math. Hons. II Year

Semestral Examination 2001-2002

Subject: Analysis III | Instructor: A. Sitaram

Date:26-11-2001 | Total Marks 115

Answer all the questions. The maximum marks you can score is 100.

1. Decide if the following statements are true or false. If true, give a proof, if false give a counter example:

   a) (X, d) is a compact metric space. If a sequence $\{f_n\}$ of continuous functions converges pointwise to a continuous function f, then necessarily $f_n \xrightarrow{u} f$. (15)

   b) Consider the series $\sum_{n=1}^{\infty} f_n(x)$. If $|f_n(x)| \leq \frac{1}{n^2}\ \forall\ n, x$, then the series converges uniformly. (15)

   c) Consider two power series $\sum_{n=0}^{\infty} a_n(z-a)^n$ and $\sum_{n=0}^{\infty} b_n(z-b)^n$. You are told that $\exists\ N$ s.t. $\forall\ n \geq N, |a_n| \leq |b_n|$. Then the radius of convergence of the first series is greater than or equal to the radius of convergence of the second series. (15)

2. Find the radius of convergence of the following series:

   a) $\sum_{n=1}^{\infty} n^{\sqrt{n}} z^n$ (10)

   b) $\sum_{n=1}^{\infty} (n!)(z-3)^n$ (10)

3. Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{(z-n)^2}, z \in \mathbb{C}, z \notin \mathbb{Z}^+.$$

   Prove that if K is a compact subset of $\mathbb{C}$ *not* intersecting $\mathbb{Z}^+$, then the series converges uniformly, for $z \in K$. (20)

4. f is a periodic function of period 2π in the class ζ with Fourier series given by $\sum_{n=-\infty}^{\infty} a_n e^{inx}$. Prove that the Fourier series $\sum_{n=-\infty}^{\infty} a_n^2 e^{inx}$ converges to a continuous function. (10)

5. Consider the Fourier series of the function $f(x) = x, -\pi \leq x < \pi$ (and repeated periodically). Find the value of the sum of squares of the absolute value of the Fourier coefficients. (10)

6. (a) Compute the curvature, torsion and Frenet frame at $t \in R$ for the *helix* given by

$$\begin{aligned} c \; : \; & R \rightarrow R^3 \\ & t \mapsto \frac{1}{\sqrt{2}}(\cos t, \sin t, t) \end{aligned}$$

(b) Compute the Weingarten map, the principal, mean and scalar curvatures of the top sheet of the hyperboloid of 2 sheets, which is defined by

$$X = \{(X_1, X_2, X_3) \in R^3 : X_3^2 - X_1^2 - X_2^2 = 1, X_3 > 0\}$$

(10)